

**Fourth Semester B.E. Degree Examination, December 2011**  
**Engineering Mathematics – IV**

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions, selecting at least TWO questions from each part.**

**PART – A**

- 1 a. Given  $\frac{dy}{dx} = \frac{1}{1+x^2} - 2y^2$ ,  $y(0) = 0$ . Find  $y(0.5)$  in two steps, using the modified Euler's method. (07 Marks)
- b. Using the Runge-Kutta method of fourth order find  $y(0.2)$  for the equation  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ,  $y(0) = 1$ , taking  $h = 0.1$ . (07 Marks)
- c. Given  $2\frac{dy}{dx} = (1+x^2)y^2$  and  $y(0) = 1$ ,  $y(0.1) = 1.06$ ,  $y(0.2) = 1.12$ ,  $y(0.3) = 1.21$ . Evaluate  $y(0.4)$  by Milne's method. (06 Marks)
- 2 a. Obtain the necessary conditions in the Cartesian system, for a function  $f(z)$  to be analytic in a region R. (07 Marks)
- b. Find the analytic function  $f(z) = u + iv$ , given  $u - v = e^x (\cos y - \sin y)$ . (07 Marks)
- c. Find the bilinear transformation that maps the points 0, -i, -1 of z-plane onto the points i, 1, 0 of w-plane respectively. (06 Marks)
- 3 a. State and prove Cauchy's integral formula. (07 Marks)
- b. Obtain the power series which represents the function  $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ , in the following regions: i)  $|z| < 2$  ii)  $2 < |z| < 3$  iii)  $|z| > 3$  (07 Marks)
- c. Using the Cauchy's residue theorem, evaluate the integral  $\int_c \frac{z^2}{(z-1)^2(z+2)} dz$ , where c is the circle  $|z| = 5/2$ . (06 Marks)
- 4 a. Solve in series the equation,  $\frac{d^2y}{dx^2} + x^2y = 0$ . (07 Marks)
- b. Solve the Bessel's equation of order n given by,  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$  where n is a non-negative real constant. (07 Marks)
- c. With the usual notations, show that  

$$x^4 - 3x^2 + x = \frac{8}{35}P_4(x) - \frac{10}{7}P_2(x) + P_1(x) - \frac{4}{5}P_0(x)$$
 (06 Marks)

**PART – B**

- 5 a. The pressure and volume of a gas are related by the equation  $PV^\gamma = k$ ,  $\gamma$  and k being constants. Fit this equation for the following set of observations:

P(kg/cm <sup>2</sup> )	0.5	1.0	1.5	2.0	2.5	3.0
V(litres)	1.62	1.00	0.75	0.62	0.52	0.46

(07 Marks)

- b. While calculating correlation coefficient between two variables x and y from 25 pairs of observations, the following results were obtained:

$$n = 25, \Sigma x = 125, \Sigma x^2 = 650, \Sigma y = 100, \Sigma y^2 = 460, \Sigma xy = 508.$$

Later it was discovered at the time of checking that the pairs of values.

x	y
8	12
6	8

were copied down as

x	y
6	14
8	6

Obtain the correct value of correlation coefficient. (07 Marks)

- c. A box contains 500 IC chips of which 100 are manufactured by company X and the rest by company Y. It is estimated that 10% of the chips made by company X and 5% made by company Y are defective. If a randomly selected chip is found to be defective, find the probability that it came from company X. (06 Marks)

- 6 a. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of successes. (07 Marks)

- b. For the Poisson distribution, prove that,  $P(r) = \frac{e^{-m} m^r}{r!}$ , where m is the mean of distribution. (07 Marks)

- c. Fit a normal distribution to the following data:

x:	1	3	5	7	9
y:	2	2	3	2	1

(06 Marks)

- 7 a. Explain the meanings of i) Null hypothesis type-I and type-II errors ii) Level of significance. (07 Marks)

- b. Eleven school boys were given a test in drawing. They were given months further tuition and a second test of equal difficulty was held at the end of it. Do the following marks give evidence that the students have benefited by extra coaching? (07 Marks)

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks I test	23	20	19	21	18	20	18	17	23	16	19
Marks II test	24	19	22	18	20	22	20	20	23	20	17

- c. A survey of 64 families with 3 children each is conducted and the number of male children in each family is noted. The results are tabulated as follows:

Male children	0	1	2	3	Total
Families	6	19	29	10	64

Apply Chi-square test of goodness of fit to test whether male and female children are equiprobable. (06 Marks)

- 8 a. Compute i)  $P(x = 1, y = 2)$  ii)  $P(x \geq 1, y \leq 2)$  iii)  $P(x \leq 1, y \leq 2)$  iv)  $P(x + y \geq 2)$ , using the following joint probability distribution for x and y. (07 Marks)

	y	0	1	2	3	Sum
x	0	0	1/8	1/4	1/8	1/2
1		1/8	1/4	1/8	0	1/2
Sum		1/8	3/8	3/8	1/8	1

- b. Discuss : i) Absorbing state ii) Transient state iii) Recurrent state iv) Periodic state. (07 Marks)

- c. A software engineer goes to his work place every day by motor bike or by car. He never goes by bike on two consecutive days but if he goes by car on a day then he is equally likely to go by car or by bike on the next day. Find the transition matrix for the chain of the mode of transport he uses. If car is used on the first day of week, find the probability that i) bike is used ii) car is used on the fifth day. (06 Marks)

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